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Canonical correlation analysis of hydrological response and soil erosion under moving rainfall*

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Abstract: The impacts of rainfall direction on the degree of hydrological response to rainfall properties were investigated using comparative rainfall-runoff experiments on a small-scale slope (4 m×1 m), as well as canonical correlation analysis (CCA). The results of the CCA, based on the observed data showed that, under conditions of both upstream and downstream rainfall movements, the hydrological process can be divided into instantaneous and cumulative responses, for which the driving forces are rainfall intensity and total rainfall, and coupling with splash erosion and wash erosion, respectively. The response of peak runoff (P_r) to intensity-dominated rainfall action appeared to be the most significant, and also runoff (R) to rainfall-dominated action, both for upstream- and downstream-moving conditions. Furthermore, the responses of sediment erosion in downstream-moving condition were more significant than those in upstream-moving condition. This study indicated that a CCA between rainfall and hydrological characteristics is effective for further exploring the rainfall-runoff-erosion mechanism under conditions of moving rainfall, especially for the downstream movement condition.

Key words: Moving rainfall, Runoff, Sediment erosion, Canonical correlation analysis (CCA)
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1 Introduction

Rainfall-runoff processes, together with the related upland erosion and sediment transport, are highly complex, and are impacted by two main aspects: rainfall and watershed characteristics (Yen and Chow, 1969; Singh, 2002; de Lima and Singh, 2003; Nunes *et al*., 2006; de Lima *et al*., 2009; Ran *et al*., 2012a; 2012b; Seo *et al*., 2012). Watershed characteristics usually include topography, shape, slope, drainage pattern, etc. (Montgomery and Dietrich, 2002; Assouline and Ben-Hur, 2006; Ran *et al*., 2009; Seo and Schmidt, 2012). Rainfall characteristics, including rainfall intensity, duration, direction, and velocity of movement, are more variable, in spatial and temporal contexts, and often impact both the integrated response (e.g., runoff hydrograph) (de Lima *et al*., 2009; Seo *et al*., 2012) and the distributed response (e.g., the temporal and spatial variability of soil moisture) (Ran *et al*., 2009; 2012a).

As characteristics of natural rainfall, directions of rainfall movement and velocity have an important influence on the runoff response and soil loss (Seo and Schmidt, 2012). Singh (2002) pointed out that the rainfall movement velocity has a significant influence on the surface/near-surface hydrologic response and soil erosion, especially for extreme storms. Storms that move rapidly have much less impact on peak discharge than those moving at an equal speed. Yen and Chow (1968) showed that a lower velocity causes a larger peak discharge, and less time reaches peak. While given the impacts by soil properties, runoff is also generated as a result of crust development on the soil surface during or after precipitation (Ran *et al*., 2012b). Crusts always affect runoff generation by

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decreasing surface K_s and subsequently also water infiltration (Carmi and Berliner, 2008). Previous studies have shown that ignoring storm movement can result in considerable overestimation and underestimation of runoff peaks (de Lima and Singh, 2003; Seo and Schmidt., 2012). de Lima *et al.* (2009) simulated storms crossing in different directions, and showed that soil loss resulting from rainstorms moving in different directions were clearly linked to the characteristics of the corresponding overland flow hydrographs and peak discharge. Seo and Schmidt (2012) also studied the relations among the direction of rainfall movement, the maximum peak discharge, and the network configuration. However, still few studies have revealed the impacts of rainfall movement directions on the characteristics of different runoff and erosion development stages, coupled with the crust development properties during those periods.

The influences of rainfall characteristics on hydrologic responses have been investigated experimentally (de Lima *et al.*, 2009; Ran *et al.*, 2011; 2012b; 2012c; He *et al.*, 2013), using field investigations (Ran *et al.*, 2011), and through computational modeling (Singh, 2005; Nunes *et al.*, 2006; Ran *et al.*, 2009). Olson and Wischmeier (1963) measured the soil loss per unit of rainfall erosivity based on simulated rainfall and plot experiments, and then scientists led by Wischmeier developed the famous universal soil loss equation (USLE), in which rainfall was considered a major influencing factor impacting overflow (Foster *et al.*, 1977; Wischmeier and Smith, 1978). Since then, quantitative studies of the effects of rainfall characteristics on soil erosion have been conducted (Nunes *et al.*, 2006; Ran *et al.*, 2012a; 2012b). Dessu and Melesse (2012) recently found that the soil and water assessment tool (SWAT) has the potential to simulate the long-term rainfall runoff process. Meanwhile, some scholars have begun to further discuss the mechanisms of hydrological response using statistical methods (e.g., SPSS), again in order to systematically study the relationships between the response factors (Rice, 1972; Arthur *et al.*, 2011; Pappas *et al.*, 2011), while still remaining at the macro level, and cannot reveal the detail internal correlations between rainfall and slope response properties. Furthermore, directions of rainfall movement are always ignored in those statistical methods.

The objective of this study is to discuss the canonical correlation between sets of rainfall characteristics and hydrological response characteristics by systematic statistical methods, and to compare the response degree under different directions of rainfall movement along a small-scale slope.

2 Materials and methods

2.1 Laboratory experiments

The experiments carried out in this study involved the use of a rainfall simulator, a tilted soil flume, a runoff recording system, and a set of soil water content monitoring devices, set up as shown in Fig. 1. Multiple scenarios relating to various rainfall movement directions, intensities (I_r) and event durations (D_r) were considered. Details of the experimental facilities (e.g., rainfall simulator, soil flume, and gathering devices) as well as the initial treatment of the soil can be found in descriptions in our previous study (Ran *et al.*, 2012b).

The rainfall scenarios used in this study were constructed by varying four parameters: direction of rainfall movement, rainfall intensity (I_r), duration (D_r), and the interval between rainfall events. To simplify the laboratory experiments, only two directions were considered for rainfall movement: upstream and downstream, with rainfall moving in only one direction during each event. Three combinations of I_r and D_r were considered: low I_r (1×10^{-5} m/s) with long D_r (120–240 min), moderate I_r (2.5×10^{-5} m/s) with medium D_r (60–120 min), and extreme I_r (4×10^{-5} m/s) with short D_r (15–60 min). The raindrop generator was moved upstream or downstream by 0.2 m at regular time intervals for each moving rainfall scenario. The equivalent velocity for this movement ranged from 0.2×10^{-3} m/s to 4.3×10^{-3} m/s. Details for setting, as well as the data used in the experiments were described in our previous study (Ran *et al.*, 2012b).

2.2 Canonical correlation analysis (CCA)

Canonical correlation analysis (CCA), a method for studying the correlativity between two different sets of variables, is aimed to identify and quantify their internal relationship. The brief mathematical principles of CCA are presented as follows (Wang *et al.*, 2012).

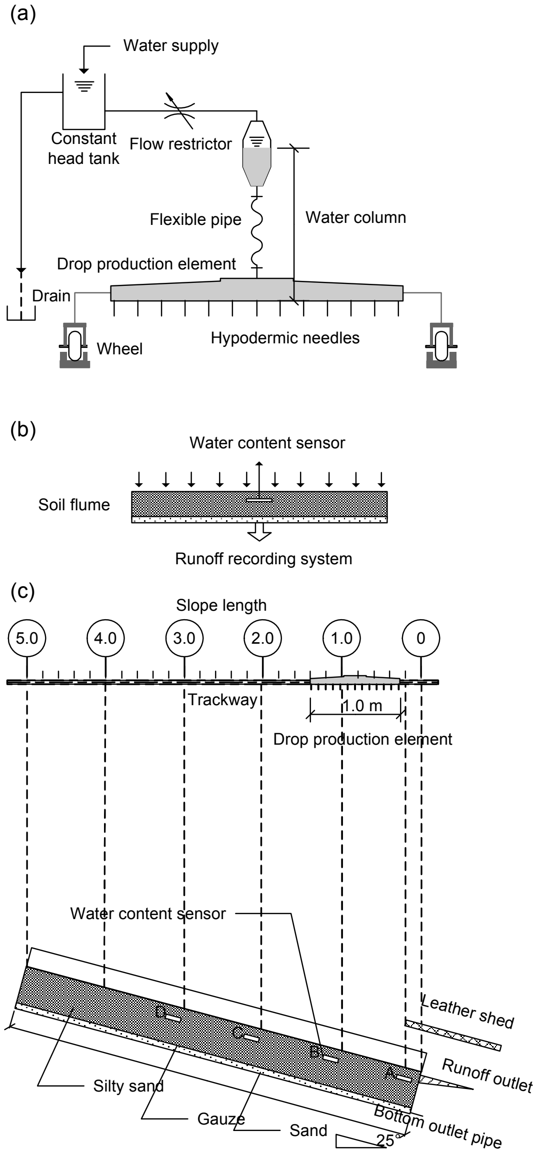

Fig. 1 Schematic representation of the laboratory experimental set-up used in this study
(a) Section drawing of the experimental set-up; (b) Section drawing of the soil flume at the outlet; (c) Elevation drawing of the experimental set-up (unit: m)

Two sets of vectors, $\boldsymbol{X}=[x_1, x_2, ..., x_p]$, $\boldsymbol{X}\in\mathbb{R}^{N\times p}$, and $\boldsymbol{Y}=[y_1, y_2,..., y_q]$, $\boldsymbol{Y}\in\mathbb{R}^{N\times q}$, and their linear combinations $\boldsymbol{U}=\boldsymbol{a}^{\mathrm{T}}\boldsymbol{X}$ and $\boldsymbol{V}=\boldsymbol{b}^{\mathrm{T}}\boldsymbol{Y}$, were used to study the correlativity between the primitive variables $\boldsymbol{X}$ and $\boldsymbol{Y}$.

CCA seeks a pair of vectors, $\boldsymbol{a}$ and $\boldsymbol{b}$, which maximize the correlation $\rho(\boldsymbol{U}, \boldsymbol{V})$,

$$\rho(\boldsymbol{U},\boldsymbol{V})=\frac{\mathrm{Cov}(\boldsymbol{U},\boldsymbol{V})}{\sqrt{\mathrm{Var}(\boldsymbol{U})}\sqrt{\mathrm{Var}(\boldsymbol{V})}}=\frac{\boldsymbol{a}^{\mathrm{T}}\boldsymbol{\Sigma}_{12}\boldsymbol{b}}{\sqrt{\boldsymbol{a}^{\mathrm{T}}\boldsymbol{\Sigma}_{11}\boldsymbol{a}}\sqrt{\boldsymbol{b}^{\mathrm{T}}\boldsymbol{\Sigma}_{22}\boldsymbol{b}}}, \tag{1}$$

where $\boldsymbol{\Sigma}_{12}$ is a sample covariance matrix between $\boldsymbol{X}$ and $\boldsymbol{Y}$, $\boldsymbol{\Sigma}_{21}$ between $\boldsymbol{Y}$ and $\boldsymbol{X}$, and $\boldsymbol{\Sigma}_{12}=\boldsymbol{\Sigma}_{21}^{\mathrm{T}}$; $\boldsymbol{\Sigma}_{11}$ and $\boldsymbol{\Sigma}_{22}$ are the covariance matrices of $\boldsymbol{X}$ and $\boldsymbol{Y}$, respectively.

The correlation coefficients of these random variables do not change if they are multiplied by a constant; some constraints (Eq. (2)) are included in Eq. (1) in order to prevent unnecessary repetition:

$$\mathrm{Var}(\boldsymbol{U})=\boldsymbol{a}^{\mathrm{T}}\boldsymbol{\Sigma}_{11}\boldsymbol{a}=1,\ \mathrm{Var}(\boldsymbol{V})=\boldsymbol{b}^{\mathrm{T}}\boldsymbol{\Sigma}_{11}\boldsymbol{b}=1. \tag{2}$$

Setting $\boldsymbol{A}=\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}$ and $\boldsymbol{B}=\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}$, the solution of Eq. (1) can be obtained by solving either of the following two eigenvalue problems:

$$\boldsymbol{A}\boldsymbol{a}=\lambda^2\boldsymbol{a},\ \boldsymbol{B}\boldsymbol{b}=\lambda^2\boldsymbol{b}, \tag{3}$$

where the square roots of the eigenvalues λ^2, obtained from Eq. (3), are called canonical correlations, and the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are the eigenvectors corresponding to $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively. Consequently, we acquire the ith set of canonical variables:

$$\boldsymbol{U}_i=\boldsymbol{a}^{(i)\mathrm{T}}\boldsymbol{X},\ \boldsymbol{V}_i=\boldsymbol{b}^{(i)\mathrm{T}}\boldsymbol{Y},\ i=1, 2, \ldots, p, \tag{4}$$

as well as the ith canonical correlation coefficient λ.

3 Results

In this study, a total of 65 1-d laboratory experiments, comprising 33 upstream-moving and 32 downstream-moving rainfall events, were carried out in 23 d. Table 1 summarizes the results obtained from the physical experiments. Table 1 includes two sets of information from the experiment data: rainfall and slope response (runoff, erosion). Therefore, two sets of primitive variables for the CCA are presented as follows:

1. Primitive variables of rainfall characteristics:

$$\boldsymbol{X}=(D_{\mathrm{r}}, I_{\mathrm{r}}, Q)^{\mathrm{T}},$$

where Q is the total rainfall for each experiment.

2. Primitive variables of hydrological response characteristics:

Table 1 Experimental observed results in the study for upstream- and downstream-moving conditions, respectively

Serial number	Test date	Test number	No-rain interval (s)	P_r ($\times10^{-6}$ m^3/s)	R ($\times10^{-6}$ m^3)	C_s (g/L)	P_s (g/s)	S (g)
Upstream-moving								
1	08-13	US001		30.2	22 088	57.1392	2.1800	1262.0902
2		US002	850	38.8	31 002	30.5435	1.5950	946.9093
3		US003	1750	40.0	31 898	20.0078	1.0200	638.2087
4		US004	3550	38.9	31 953	14.6201	0.7330	467.1559
5		US005	12 250	40.8	31 635	10.8623	0.6140	343.6231
6	06-27	US006		24.3	36 147	57.4379	1.4795	2076.1852
7		US007	1700	28.8	45 929	31.7919	1.0000	1460.1692
8		US008	3500	27.5	44 425	19.4827	0.6505	865.5200
9		US009	10 700	23.8	38 085	12.4460	0.4505	474.0071
10	08-21	US010		44.1	137 000	19.9227	0.9440	2729.4038
11		US011	3400	49.6	159 500	8.8707	0.5475	1414.8714
12		US012	11 500	48.5	159 300	6.2134	0.3585	989.7924
13	07-27	US013		43.3	274 680	30.2731	1.2753	8315.4166
14		US014	14 000	45.0	283 553	15.8695	0.7437	4499.8510
15		UM001	3380	13.6	16 780	69.2465	0.9965	1161.9565
16		UM002	1700	22.0	34 124	36.8433	0.9275	1257.2402
17	07-25	UM003		22.4	35 071	24.2902	0.7085	851.8823
18		UM004	12 500	21.5	34 635	16.9290	0.5005	586.3376
19	06-25	UM005		14.3	40 698	29.0862	0.5013	1183.7500
20		UM006	22 200	15.3	46 202	18.1655	0.3777	839.2833
21	07-03	UM008		22.0	139 270	36.9969	0.8758	5152.5629
22		UM009	11 300	21.3	137 850	14.9373	0.3293	2059.1092
23	08-04	UM010		26.0	239 279	32.6173	0.9820	7804.6448
24		UM011	9000	27.3	268 464	22.7816	0.7195	6116.0404
25	06-21	UW001		0.0	0	0.0000	0.0000	0.0000
26		UW002	3400	0.2	154	4.6753	0.0013	0.7200
27		UW003	14 200	7.5	18 446	6.7196	0.0686	123.9500
28	06-15	UW004		0.0	0	0.0000	0.0000	0.0000
29		UW005	8700	4.2	12 571	10.6913	0.0513	134.4000
30	08-08	UW006		9.5	87 500	39.3379	0.4008	3442.0670
31		UW007	9900	10.0	95 500	24.9994	0.3022	2387.4386
32	08-17	UW008		10.8	130 644	41.0484	0.4652	5362.7320
33		UW009	7700	10.8	138 506	23.6539	0.2995	3276.2119
Downstream-moving								
34	08-15	DS001		29.0	13 643	79.5117	2.1290	1084.7785
35		DS002	850	38.9	27 896	34.5689	1.4430	964.3332
36		DS003	1750	47.3	29 921	21.8798	1.1060	654.6669
37		DS004	3550	41.5	30 100	15.9385	0.8010	479.7477
38		DS005	11 650	43.0	30 726	10.5380	0.5440	323.7915
39	06-29	DS006		24.1	21 109	40.9603	1.1595	864.6310
40		DS007	1900	27.3	41 261	19.5182	0.6170	805.3391
41		DS008	4100	27.5	41576	12.2782	0.4150	510.4771
42		DS009	3500	27.8	40 082	8.3786	0.2995	335.8327
43	08-02	DS010		39.9	92 813	49.4416	1.9830	4588.8208
44		DS011	3400	43.6	133 924	21.3428	1.1515	2858.3078
45		DS012	10 600	42.3	131 230	17.1447	1.0670	2249.9005
46	07-29	DS013		31.9	157 332	43.0180	1.3260	6768.1027
47		DS014	13 880	33.2	203 942	20.9780	0.9073	4278.2919

To be continued

Table 1

48	07-01	DM001		21.8	21757	59.2334	1.3275	1288.7402
49		DM002	1700	24.7	37425	25.4327	0.7715	951.8205
50		DM003	3500	24.5	37426	15.6115	0.4910	584.2754
51		DM004	11900	23.3	34848	10.2470	0.3325	357.0881
52	07-21	DM005		20.1	34180	64.7911	1.4627	2214.5606
53		DM006	3600	20.2	62027	31.8704	0.8443	1976.8254
54		DM007	11800	19.7	60074	20.9033	0.5263	1255.7441
55	07-05	DM008		21.2	77472	29.2863	0.7062	2268.8703
56		DM009	11900	21.8	132089	14.2250	0.4022	1878.9665
57	08-06	DM010		22.3	152310	27.7122	0.6617	4220.8460
58		DM011	9000	23.6	172201	21.0547	0.5575	3625.6409
59	06-23	DW001		1.1	730	14.0137	0.0176	10.2300
60		DW002	16000	5.5	11589	21.0640	0.1808	244.1000
61		DW003	3390	6.1	14552	16.2507	0.1475	236.4800
62	08-10	DW006		10.9	55166	40.9443	0.4577	2258.7356
63		DW007	10020	11.2	103494	24.0305	0.3160	2487.0143
64	08-19	DW008		9.9	51286		0.3720	1710.2001
65		DW009	9060	10.4	100958		0.2872	2475.9238

$$\boldsymbol{Y}=(P_\mathrm{r}, R, C_\mathrm{s}, P_\mathrm{s}, S)^\mathrm{T},$$

where P_r is the peak runoff, R the total runoff, C_s the sediment concentration, P_s the peak sediment discharge, and S the total sediment discharge at the outlet of the soil slope for each rainfall experiment.

The CCAs were conducted using SPSS based on the observed data (Table 1); the results showed that two sets of canonical relationships (I, II) between rainfall characteristics and response characteristics were obtained, for both conditions (Tables 2–6), these being: set I (CVU I) and set II (CVU II), canonical variables of the upstream movement condition; and set I (CVD I) and set II (CVD II), canonical variables of the downstream movement condition. Eqs. (5)–(8) represent the canonical conversion relations between the primitive and canonical variables. Figs. 2a–2d present the canonical loading relationships corresponding to Eqs. (5)–(8), respectively.

$$\begin{cases}\boldsymbol{U}_{\mathrm{u}1}=-0.038D_\mathrm{r}-0.631I_\mathrm{r}+1.021Q,\\ \boldsymbol{V}_{\mathrm{u}1}=-0.613P_\mathrm{r}+0.937R+0.005C_\mathrm{s}-0.116P_\mathrm{s}+0.144S,\end{cases} \tag{5}$$

$$\begin{cases}\boldsymbol{U}_{\mathrm{u}2}=0.045D_\mathrm{r}+0.881I_\mathrm{r}+0.297Q,\\ \boldsymbol{V}_{\mathrm{u}2}=0.774P_\mathrm{r}+0.326R+0.009C_\mathrm{s}+0.112P_\mathrm{s}-0.061S,\end{cases} \tag{6}$$

$$\begin{cases}\boldsymbol{U}_{\mathrm{d}1}=0.149D_\mathrm{r}+1.055I_\mathrm{r}-0.363Q,\\ \boldsymbol{V}_{\mathrm{d}1}=0.788P_\mathrm{r}-0.080R-0.095C_\mathrm{s}+0.369P_\mathrm{s}-0.203S,\end{cases} \tag{7}$$

$$\begin{cases}\boldsymbol{U}_{\mathrm{d}2}=-0.171D_\mathrm{r}+0.125I_\mathrm{r}+1.100Q,\\ \boldsymbol{V}_{\mathrm{d}2}=0.249P_\mathrm{r}+0.691R+0.306C_\mathrm{s}-0.348P_\mathrm{s}+0.345S.\end{cases} \tag{8}$$

Generally, the canonical correlation coefficient is close to 1 (not less than 0.96) for all pairs of canonical variables (Tables 2–5), clearly indicating a strong correlation between rainfall characteristics and hydrologic response characteristics.

3.1 Analysis of CVU I, CVU II, CVD I and CVD II

For CVU I (Table 2, Eq. (5), Fig. 2a (p.359)), the conversion coefficient of Q (1.021) in $U_{\mathrm{u}1}$ is much larger than that of D_r (−0.038) and I_r (−0.631), which means $\boldsymbol{U}_{\mathrm{u}1}$ mainly represents the properties of total rainfall. Here, D_r is perceived as a rectified variable, the reason is that its canonical conversion coefficient (−0.038) and canonical correlation coefficient (0.881) are opposite in sign, as is C_s in $\boldsymbol{V}_{\mathrm{u}1}$. Similarly, D_r and S in Eq. (6), D_r and C_s in Eq. (7), as well as D_r and P_s in Eq. (8) are all rectified variables for the same reason. The opposite canonical variable $\boldsymbol{V}_{\mathrm{u}1}$ mainly represents the property of R, for its largest canonical conversion coefficient (0.937), and the absolute of the coefficient for P_r is a little smaller than that of R, while they are opposite in sign. Generally, the degree of response of sediment erosion is weaker than runoff processes for the relatively small canonical conversion coefficients of P_s and S, so it is included with the canonical loading diagram (Fig. 2a).

For CVU II (Table 3, Eq. (6), Fig. 2b), it is apparent that the canonical conversion coefficient of I_r is the largest in the conversion relationship with $\boldsymbol{U}_{u2}$, which means that the whole rainfall properties appear to be more significant as I_r increases, and the coefficient of Q is much smaller than that of I_r. The opposite $\boldsymbol{V}_{u2}$, is mainly embodied by the characteristics of P_r, of which the canonical conversion coefficient is the largest (0.774), and that of R the second largest (0.326). In general, the responses of C_s and P_s are much weaker than both P_r and R in terms of their relatively small canonical coefficients. Thus, it can be seen that CVU II represents the main response characteristics of P_r under a rainfall force dominated by I_r, and also some of the weaker responses of soil erosion, so it is included with the canonical loading diagram (Fig. 2b).

As for CVD I (Table 4, Eq. (7), Fig. 2c), similar to CVU II, the absolute canonical conversion coefficient for the $\boldsymbol{U}_{d1}$ of I_r (1.055) is much larger than that for Q (0.363). P_r also has the largest effect on $\boldsymbol{V}_{d1}$ as it has the largest canonical coefficient (0.788), and P_s is the second largest (0.369), while those of R and S are clearly much weaker. Thus, CVD I mainly shows the P_r-dominated response under I_r-based rainfall action, coupled with the relatively weak response of P_s, so it is included with the canonical loading diagram (Fig. 2c).

Moreover, like CVU I, CVD II (Table 5, Eq. (8), Fig. 2d) mainly shows an R-dominated response under the main force of Q, while the responses of P_r, C_s and S respond to a similar degree.

3.2 Redundancy analysis

Table 6 shows the redundancy analysis results, presenting the reference value of the CCA. When rainfall moves upstream, the degree of explanation of $\boldsymbol{X}$—the primitive variable for rainfall properties—given by the canonical variable $\boldsymbol{U}_{u1}$, is 51.4% (CV*X*1-1), and for $\boldsymbol{U}_{u2}$ 41.8% (CV*X*1-2), $\boldsymbol{V}_{u1}$ (CV*X*2-1) 51.1%, and $\boldsymbol{V}_{u2}$ 40.8% (CV*X*2-2). However, the effect of rainfall on slope response is single-directional and their opposite canonical variables $\boldsymbol{U}_{u1}$ 26.7% (CV*Y*2-1), and $\boldsymbol{U}_{u2}$ 36.4% (CV*Y*2-2). When rainfall moves downstream, the degree of explanation of $\boldsymbol{X}$ given by $\boldsymbol{U}_{d1}$ and $\boldsymbol{U}_{d2}$ is 46.9% (CV*X*1-1) and 45.7% (CV*X*1-2), respectively, also that of $\boldsymbol{Y}$ given by $\boldsymbol{V}_{d1}$ and $\boldsymbol{V}_{d2}$ is 27.8% (CV*Y*1-1) and 38.2% (CV*Y*1-2), respectively, and $\boldsymbol{U}_{d1}$ 27.2% (CV*Y*2-1), $\boldsymbol{U}_{d2}$ 35.9% (CV*Y*2-2).

Thus, it is acceptable for the explanation degree of primitive rainfall property variables given by $\boldsymbol{U}_{u1}$ and $\boldsymbol{U}_{u2}$ for upstream-movement conditions; although it is a little poorer for that of $\boldsymbol{Y}$, because the ideal state

Table 2 Canonical correlation analysis result for CVU I

CVU I	Variable	Standard canonical coefficient	Canonical loadings	Cross loadings
$\boldsymbol{U}_{u1}$	D_r	−0.038	0.881	0.878
	I_r	−0.631	−0.320	−0.319
	Q	1.021	0.815	0.812
$\boldsymbol{V}_{u1}$	P_r	−0.613	−0.219	−0.218
	R	0.937	0.767	0.765
	C_s	0.005	−0.018	−0.018
	P_s	−0.116	−0.291	−0.290
	S	0.144	0.789	0.786

Canonical correlation between $\boldsymbol{U}_{u1}$ and $\boldsymbol{V}_{u1}$: 0.997

Table 3 Canonical correlation analysis result for CVU II

CVU II	Variable	Standard canonical coefficient	Canonical loadings	Cross loadings
$\boldsymbol{U}_{u2}$	D_r	0.045	−0.145	−0.143
	I_r	0.881	0.947	0.936
	Q	0.297	0.580	0.573
$\boldsymbol{V}_{u2}$	P_r	0.774	0.971	0.960
	R	0.326	0.641	0.633
	C_s	0.009	0.095	0.094
	P_s	0.112	0.568	0.561
	S	−0.061	0.420	0.415

Canonical correlation between $\boldsymbol{U}_{u2}$ and $\boldsymbol{V}_{u2}$: 0.988

Table 4 Canonical correlation analysis result for CVD I

CVD I	Variable	Standard canonical coefficient	Canonical loadings	Cross loadings
$\boldsymbol{U}_{d1}$	D_r	0.149	−0.652	−0.644
	I_r	1.055	0.967	0.956
	Q	−0.363	−0.211	−0.209
$\boldsymbol{V}_{d1}$	P_r	0.788	0.954	0.943
	R	−0.080	−0.043	−0.043
	C_s	−0.095	0.128	0.127
	P_s	0.369	0.679	0.671
	S	−0.203	−0.030	−0.030

Canonical correlation between $\boldsymbol{U}_{d1}$ and $\boldsymbol{V}_{d1}$: 0.988

Table 5 Canonical correlation analysis result for CVD II

CVD II	Variable	Standard canonical coefficient	Canonical loadings	Cross loadings
$\boldsymbol{U}_{u2}$	D_r	−0.171	0.600	0.582
	I_r	0.125	0.250	0.242
	Q	1.100	0.974	0.944
$\boldsymbol{V}_{u2}$	P_r	0.249	0.248	0.240
	R	0.691	0.973	0.943
	C_s	0.306	0.063	0.061
	P_s	−0.348	0.208	0.207
	S	0.345	0.925	0.897

Canonical correlation between $\boldsymbol{U}_{d2}$ and $\boldsymbol{V}_{d2}$: 0.969

of the degree of explanation should not be less than 30%, in theory (Zhang, 2004), it might be perceived as more or less reluctantly accepted. When rainfall moves downstream, apart from the weaker degree of explanation of $\boldsymbol{Y}$ given by its set I of canonical variables $\boldsymbol{U}_{\text{d1}}$ and $\boldsymbol{V}_{\text{d1}}$, other degrees of explanation for canonical variables are generally acceptable.

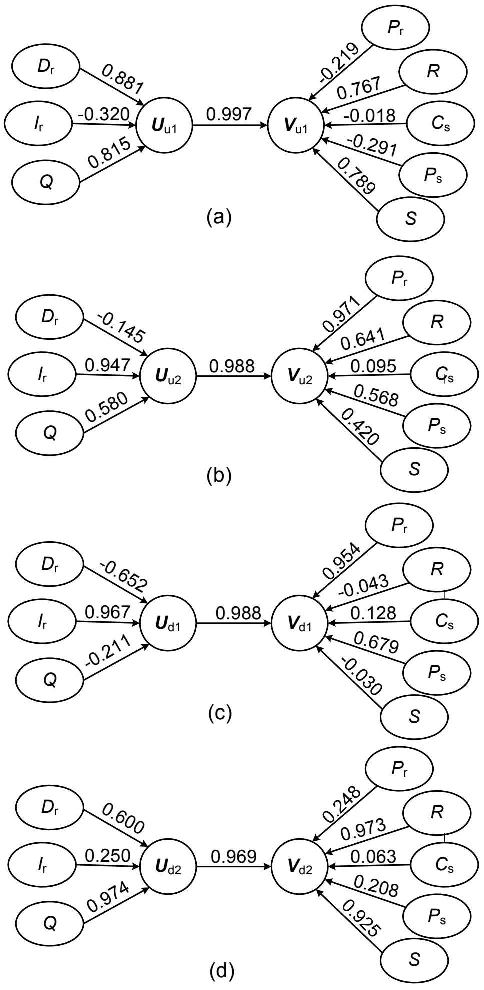

Fig. 2 Canonical loadings between the observed and canonical variables CVU I (a), CVU II (b), CVD I (c) and CVD II (d)

4 Discussion

Based on the CCA between the variables of rainfall and hydrologic response, it can be seen that, for both rainfall directions, the rainfall-runoff-erosion process in this study can be divided into an instantaneous and a cumulative response, of which the driving force is rainfall intensity and total rainfall, respectively, accompanied by splash erosion and wash erosion for each response.

When rainfall moves upstream, the canonical conversion coefficients of R and P_{r} have opposite signs for the cumulative response (Eq. (5)), which means the slope response appears stronger with increasing R, while the decrease in P_{r}, occurs mainly because D_{r} in the experiments with weak I_{r} is much longer than those with strong I_{r}, which leads to the much larger Q of the former. Moreover, P_{r} is mostly determined by rainfall intensity because the slope responses are already in steady state when P_{r} emerges (Ran *et al.*, 2012b); hence, the larger the Q, the smaller the I_{r}, leading to the weaker response of P_{r}, just the opposite trend to the runoff response. When rainfall moves downstream, the canonical conversion coefficients of R and P_{r} have the same sign, which means the slope hydrologic response appears stronger with increases of both R and P_{r}. Though under nearly the same initial conditions, the directions of rainfall and the overflow are the same along the slope while rainfall moves downstream. Consequently, a thin layer of sedimentary crust would form in the initial rainfall stages, due to initial overflow from the relatively high positions on the slope, resulting in evidently smaller permeability coefficient (K_{s}) and larger compaction of the surface soil than for the condition of upstream movement (Robinson and Woodun, 2008) together with the cumulative washing by overflow before peak runoff emerges. Thus, peak runoff appears larger than that for upstream movement experiments under similar conditions (Ran *et al.*, 2012b).

Table 6 Redundancy analysis of CCA results in this study

Rainfall movement	Proportion of variance of $\boldsymbol{X}$ explained by $\boldsymbol{U}_{\text{u1}}$, $\boldsymbol{U}_{\text{u2}}$, $\boldsymbol{U}_{\text{d1}}$, $\boldsymbol{U}_{\text{d2}}$ (%)		Proportion of variance of $\boldsymbol{X}$ explained by $\boldsymbol{V}_{\text{u1}}$, $\boldsymbol{V}_{\text{u2}}$, $\boldsymbol{V}_{\text{d1}}$, $\boldsymbol{V}_{\text{d2}}$ (%)	
	CV*X*1-1	CV*X*1-2	CV*X*2-1	CV*X*2-2
Upstream-moving	51.4	41.8	51.1	40.8
Downstream-moving	46.9	45.7	45.7	43.0

Rainfall movement	Proportion of variance of $\boldsymbol{Y}$ explained by $\boldsymbol{V}_{\text{u1}}$, $\boldsymbol{V}_{\text{u2}}$, $\boldsymbol{V}_{\text{d1}}$, $\boldsymbol{V}_{\text{d2}}$ (%)		Proportion of variance of $\boldsymbol{Y}$ explained by $\boldsymbol{U}_{\text{u1}}$, $\boldsymbol{U}_{\text{u2}}$, $\boldsymbol{U}_{\text{d1}}$, $\boldsymbol{U}_{\text{d2}}$ (%)	
	CV*Y*1-1	CV*Y*1-2	CV*Y*2-1	CV*Y*2-2
Upstream-moving	26.9	37.2	26.7	36.4
Downstream-moving	27.8	38.2	27.2	35.9

With the exception of the rectified variables of $\boldsymbol{V}_{\mathrm{u1}}$ and $\boldsymbol{V}_{\mathrm{u2}}$, it is shown both in the conversion formulas (Eqs. (5) and (8)) and canonical loading diagrams (Figs. 2a and 2d) that the degrees of erosion under conditions of the two rainfall directions are significantly different. In conditions of upstream movement, both P_{s} and S have little impact on $\boldsymbol{V}_{\mathrm{u1}}$ for their significantly smaller values of absolute conversion coefficients. However, when rainfall moves downstream, both C_{s} and S (the erosion property variables) have a relatively large impact on $\boldsymbol{V}_{\mathrm{d2}}$, because gaps between their canonical conversion coefficients, as well as canonical loadings and that of R, are a lot smaller than those in conditions of upstream movement. It was also found for $\boldsymbol{V}_{\mathrm{d2}}$ that the degrees of influence of P_{r}, C_{s}, and S are very close, the reason is that the erosive sediment on the slope surface could be relatively completely washed by overflow because the rainfall and overflow move in the same direction along the slope. When rainfall moves upstream, the change of surface permeability was less than that in downstream conditions (Ran *et al.*, 2012b). In contrast, the erosive sediment could not be washed completely towards the outlet unless the washing duration was long enough, because the rainfall and overflow move in opposite directions along the slope (Ran *et al.*, 2012a). On the whole, in response to the Q-dominated rainfall action, the erosion response is more strongly indicative of the whole hydrologic response characteristics for conditions of downstream rather than upstream movement.

In response to the force which is dominated by I_{r} (rainfall action), P_{r} appears to be the best determining factor of all the response characteristics, whether for conditions of upstream or downstream movement (Eqs. (6) and (7); Figs. 2b and 2c). Compared with the canonical loadings for P_{r} and R, both C_{s} and P_{s} have little impact on $\boldsymbol{V}_{\mathrm{u2}}$ when rainfall moves upstream (Eq. (6), Fig. 2b). This was particularly so for C_{s}, a cumulative variable, the response of which could be ignored, so are the variables R and S in $\boldsymbol{V}_{\mathrm{d1}}$. Furthermore, both the canonical conversion coefficient and canonical loading of P_{r} are much larger than that for P_{s}, because as each response is made, the surface hydrologic responses are mostly in a steady state, resulting in much smaller K_{s} (Ran *et al.*, 2012b) and greater compaction of surface soil. Compared to the response at the beginning of each day, the majority of experiments appear to obviously increase in runoff levels, and have a decreased sediment discharge in this steady state (Table 1). As mentioned above, the erosive sediment may be left on the slope unless the washing time continues for a sufficient period, owing to the opposite directions of movement between rainfall and overflow. Thus, P_{r} has the strongest influence on the canonical variables of response properties, and P_{s} has a slightly weaker influence. For conditions of downstream movement, both the canonical loadings and conversion coefficient of P_{s} in $\boldsymbol{V}_{\mathrm{d1}}$ are larger than those of P_{r} (Eq. (6)). Generally, the peak sediment discharge in downstream movement experiments has a larger impact on the canonical variable of response properties than conditions of upstream movement, of which reason is identical as the cumulative response mentioned above (Eqs. (5) and (8)).

Generally, for the dominant variables $\boldsymbol{U}_{\mathrm{d1}}$ and $\boldsymbol{U}_{\mathrm{d2}}$, absolutes for both canonical loadings and canonical conversion coefficients are much larger than those for other variables, showing the most distinctive divide between instantaneous and cumulative responses. The divide is, however, not so obvious in upstream movement experiments. This means that the analytics work of CCA for the conditions of downstream movement is more effective.

5 Conclusions

Based on the comparative rainfall-runoff experiments on a small-scale slope (4 m×1 m) under conditions of different rainfall movement directions along the slope surface, as well as the CCA between the rainfall and hydrologic response characteristics, it was found that the CCA method is valid for research into hydrologic responses to rainfall movement, and several conclusions were obtained via CCA.

1. Under both rainfall directions, the rainfall -runoff-erosion process in this study can be divided into instantaneous and cumulative responses, of which the driving force is rainfall intensity and total rainfall, respectively, accompanied by splash erosion and wash erosion for each response. Rainfall duration in this study did not have a dominant effect on the hydrologic response.

2. The response of P_{r} to the I_{r}-dominated rainfall action appeared to be the most significant, both for conditions of upstream and downstream movement. The response of sediment erosion to rainfall appeared obviously weaker than that of runoff characteristics. The instantaneous responses of erosion in downstream

conditions appear more strongly than those for upstream conditions. However, in reality, those responses may be more complex for various impact factors in nature, future studies will be carried out to further explore this subject.

3. The response of R to the Q-dominated rainfall action appeared to be the most significant for conditions of both upstream and downstream movements. The cumulative responses of sediment erosion in downstream movement were also more significant than those for upstream movement.

4. The analytics work by CCA is more effective for the condition of downstream movement than upstream movement.

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